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THE DETERMINATION OF ORBITS.

Die Bahnbestimmung der Himmelskörper. By Julius Bauschinger. Pp. xv+653; mit 84 Figuren im Text. (Leipzig: Wilhelm Engelmann, 1906.)

Die Gauss-Gibbssche Methode der Bahnbestimmung eines Himmelskörpers aus drei Beobachtungen. Mit einem Anhang zum "Grundriss der theoretischen Astronomie." By Prof. Johannes Frischauf. Pp. 47. (Leipzig: Wilhelm Engelmann, 1905.)

THE development of convenient and general methods for calculating the orbit of any body around the sun from a limited number of observations constitutes a classical problem in the annals of astronomy. Its history, which has yet to be adequately written, now covers a period of rather more than two centuries, and during that time it has attracted the attention of many famous mathematicians whose successes and failures are alike remarkable. At the time of Newton long records had made the principal features of the orbits of the known planets familiar, and no addition to their number was made within the next hundred years. Hence in the eighteenth century efforts were mainly directed to the determination of the parabolic orbits of comets. Yet the completely satisfactory solution was deferred until 1797, when Olbers's celebrated work appeared. Why Olbers succeeded when far greater mathematicians, such as Euler and Lagrange, had met with comparative failure is an interesting question. The fact is that the determination of orbits is an art demanding as such a sense of arithmetical technique and not merely an insight into the mathematical principles involved.

In the nineteenth century, on the other hand, the discovery of minor planets, which are now being found at the average rate of one a week, has required general methods of dealing with planetary orbits. The deduction of an orbit from the necessary three observations has been based mainly on the methods of Gauss's "Theoria Motus." Even in matters of detail the variations which have been added have been for the most part slight and unimportant. In a less degree use has been made of the earlier method of Laplace, which has been generally regarded as inferior in practice. In reality the two solutions are essentially equivalent as regards their mathematical foundation, a remarkable theorem due to Lambert standing as the formal connecting link. Again the difference is a matter of technique rather than of principle.

The determination of orbits, considered in a wide sense, forms a subject so complicated and so closely dependent on other branches of astronomy that comprehensive treatises serving to bring together what experience has shown to be the most practical methods have rendered indispensable service. In England, owing, perhaps, to the too exclusive predominance of one school of thought, little has been contributed to the development of the theory and nothing to its connected presentment. The well-known treatise of Watson we owe to America. Of other works, by far the most notable is that of Oppolzer. Unfortunately, the second volume of this book is now out of print and has become scarce.

In these circumstances a warm welcome must be extended to Dr. Bauschinger's treatise. His position as Director of the Recheninstitut in Berlin, the prominent feature of the work of which is the surveillance of the rapidly accumulating multitude of minor planets, leads us to expect an eminently practical treatment, and we are not disappointed. No great originality will be found, nor was it to be looked for, so far as regards the fundamental methods themselves. The aim of the author has evidently been to follow the path which has been proved by experience, and any originality must be sought in the modes of presentation, which are always elegant, concise, and lucid.

A most important feature of a work of this kind is the choice of illustrative examples of actual computations. In both the liberal selection and the arrangement of these Dr. Bauschinger has done well. The diagrams are neat and clear. The style of printing, a matter of which the importance in the case of a mathematical work can hardly be exaggerated, will bear comparison with the best English examples of a similar class. It is impossible that all errors should have been detected in the course of proof-reading, but though two or three have certainly escaped notice, it is unlikely that there will be any necessity for a list of corrections such as that inserted in Oppolzer's second volume.

Some time ago Dr. Bauschinger published a very useful collection of astronomical tables. Frequent reference is made to these in the present work, which is thus relieved of a large amount of additional matter, while the tables themselves are available in a handier form than as an appendix to a bulky volume. As it is, the author has covered the same ground as Oppolzer's "Bahnbestimmung," and even included some additions within the limits of a single volume. But it is of necessity a large one, and can scarcely fail to suggest the question whether its size could not be reduced by omissions or compression without prejudice to its utility. At first sight this would certainly seem to be the case. The first part, containing a discussion of astronomical coordinates, is occupied with matter which ought to be accessible in general treatises on practical astronomy. The chapter on the method of least squares might be replaced by simple references to some work devoted to that subject, and what is given in the chapter on mechanical integration ought to be found in treatises on the calculus of finite differences. But apart from the fact that this supposes the existence of ideal books which have not yet been written, it is a distinct advantage to be saved the trouble of consulting a number of separate works, even when these are at hand. The fuller treatment must be justified by a severely concise and practical discussion of all subordinate topics, and in this respect little fault will be found with Dr. Bauschinger's handling of his material. It is difficult to believe,

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however, that some matters of elementary mathematics could not have been omitted without detriment. Thus the discussion, at the beginning of the second part, of the equations of a conic, based on the definition of a conic as the plane section of a right circular cone, must be superfluous for a reader who is capable of following the whole of the first part intelligently. But the fault is doubtless on the right side.

The whole work consists of seven parts. The first deals with those portions of general astronomy which are relevant to the main purpose. The chapters on time and on precession and nutation seem particularly clear and good. That on aberration follows the traditional lines of Gauss and Bessel, and criticism would be out of place here. Yet the exposition of Gauss, which seems to assume the apparent composition of the velocities of light and of the earth as a matter of course, appears to be imperfect in view of the difficulties in the physical theory. Is it not more logical to consider the apparent composition as an inductive result instead of the explanation of the astronomical phenomena?

The second part contains a discussion of undisturbed heliocentric motion. Dr. Bauschinger asserts (p. 170) that Lambert's equation is of little use in the case of ordinary elliptic orbits. This opinion may be disputed. It is true that the development in series is of little assistance owing to slow convergence, but in its original form the equation can be easily solved in all ordinary cases. The natural expression of the formulæ for motion in a hyperbola involves hyperbolic functions. The use of these is entirely avoided, presumably because tables of hyperbolic functions are not as a rule accessible to the computer.

The properties of the apparent or geocentric motion are discussed in the third part. Here will be found Bruns' elegant proof of the theorem of Lambert on the curvature of the apparent orbit. Incidentally it may be remarked that Lambert seems to have missed that measure of fame to which his unquestionable eminence as a mathematician entitles him.

The longest part is the fourth, in which the various methods of determining a preliminary orbit are described. An excellent feature is the compendious arrangement of the working formulæ. This part is followed by that on the adjustment of an orbit by the method of least squares. In both sections numerical examples are fully and clearly worked out.

The sixth part contains the theory of special perturbations. Three methods are given, according to which the perturbations can be calculated in the elements, or in polar or in rectangular coordinates. In the preliminary chapter, on mechanical integration, the usual German notation for interpolation formulæ is employed. It is difficult to see the advantage of this over the ordinary notation of finite differences. The last chapter of this section brings the reader to the determination of the definitive orbit.

Here the work might have ended, but Dr. Bauschinger has added a final part, in which he investigates the determination of the orbits of meteors, satellites, and double stars. These last chapters are necessarily brief, and it may be doubted whether, as regards unity of subject, their inclusion is justified. But that on satellites is certainly valuable, especially in view of recent discoveries.

The source of the numerous theorems which are met with in the work has generally been indicated, but this is not always the case. Thus the theorems on p. 184 are due to M. Radau (Bull. Astr., x. p. 11) and to Mr. Shin Hirayama (Monthly Notices, R.A.S., lxii., p. 620). Such references add greatly to the interest, but of course it is always difficult to be sure that the sources are strictly original. For instance, the proposition attributed (p. 131) to van der Kolk was, as has been recently pointed out, previously given by Whewell. There is an index at the end of the volume, but it is not so complete as it should have been. A full index of names is needed.

An outline of the method of Gibbs will be found in Dr. Bauschinger's work, but for fuller details the pamphlet of Dr. Frischauf may be consulted with advantage. The method is based on the use of a particular expression for the ratio of a triangle to the corresponding sector of an ellipse. The form is mathematically elegant and the degree of approximation is high, but it was thought to entail greater complexity in the computations, while, on the other hand, the method by itself gave little assistance when a still closer approximation proved necessary. This defect was remedied by Prof. Harzer. The modified method is described by Dr. Frischauf in a clear and interesting manner; the practical value of his account would have been enhanced by the addition of a fully worked example. The pamphlet also contains a number of supplementary notes to the author's "Grundriss der theoretischen Astronomie," a work of which a second edition appeared in 1903 after an interval of thirty-two years from its first publication. H. C. P.

INDUCTION AND CONDUCTION MOTORS.

Moteurs a Collecteur a Courants alternatifs. By Dr. F. Niethammer. Pp. 131. (Paris: *L'Éclairage Électrique*, 1906.)

THE title leads one to believe that the author is going to deal with at least all the principal types of modern alternate-current commutator motors, whereas the book is practically restricted to a consideration of the series induction and conduction motors. Shunt induction motors of the commutator type are occasionally touched upon, but all remarks concerning these must be considered as quite erroneous. Generally speaking, the number of mistakes is too great.

In chapter i. the historic part does not deal with the machines out of which those modern single-phase commutator motors have been directly evolved, which are afterwards considered more closely. The preliminary consideration of some of the types now in use is full of errors, and much prominence is given to the least important of these types. The indiscriminate use of the expression "repulsion" motor leads to the usual confusion.

In the second chapter, which is the most important in the whole book, we find the author trying to

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